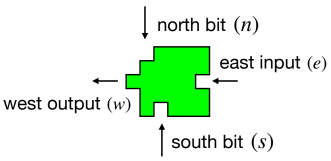
Computiles

A Geometric Binary Calculator in 10 Tiles

Can geometry itself force a computation? My son Émilien (13) and I designed *computiles*, tiles that turn binary addition and subtraction into tiling puzzles where only the correct result fits. A wrong answer literally has nowhere to go.



A *computile* is a 5×5 square tile whose geometry encodes a local binary rule. Its color shows the resulting bit $n \oplus s \oplus e$. Green for 1, red for 0.

1 Addition

Place the two binary numbers in the top and bottom rows, most significant bit on the left. Use red 0 tiles for 0 bits, green S tiles for 1s in the top row, and green N tiles for 1s in the bottom row. Then fill the middle row from right to left with the tiles in figure 1.

The tileset has eight computational cases plus two input tiles. Tile names list edges by compass direction: n north, s south, e east (incoming carry), w west (outgoing carry). Uppercase letters mark tabs, lowercase letters notches.

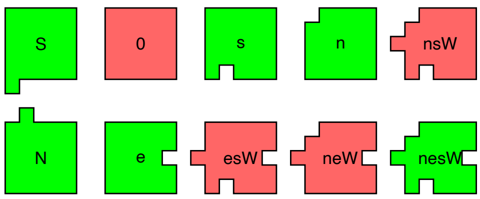


Figure 1: Tiles for binary addition:
$w = (n \cdot s) \vee (e \cdot (n \oplus s))$

Figure 2 shows $61 + 83 = 144$. At each step, exactly one tile fits. Tabs and notches are asymmetric enough to prevent accidental matches under rotation. When the row is complete, the middle line reads the sum, with any final carry appearing as an extra bit on the left.

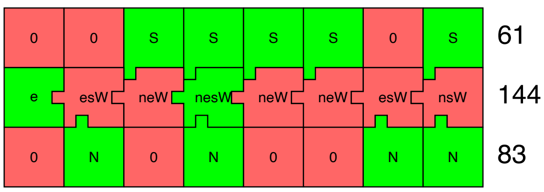


Figure 2: *Computiling* $61 + 83 = 144$.

2 Subtraction

For subtraction, build the input rows exactly as in addition, then complete the middle row using the subtraction tileset (figure 3). The east and west edges now represent the borrow signal (figure 4).

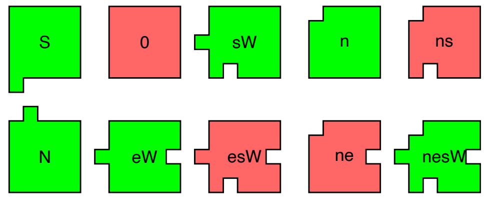


Figure 3: Tiles for binary subtraction:
$w = (\neg n \cdot s) \vee \left(e \cdot \neg(n \oplus s)\right)$

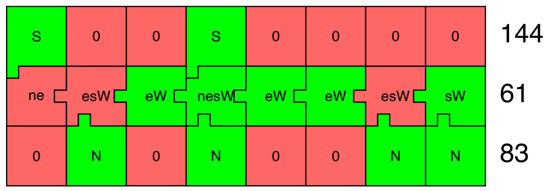


Figure 4: *Computiling* 144 - 83 = 61.

3 The Mechanics of Computiling

Wang tiles (1961) showed that edge-matching can encode computation. Computiles are a 1D specialization: a tiny finite-state machine. Each tile reads the north and south bits with the incoming east signal, displays the resulting bit in its color, and passes the updated carry or borrow westward. Together, the ten tiles implement a full adder and subtractor.

Addition and subtraction share six of the ten tiles: the two input tiles plus four computational ones. When the incoming signal e matches the bottom bit s, both operations produce the same local outcome: $w = s$ and the tile color $n \oplus s \oplus e$ reduces to n. In those four cases, one tile serves both operations (table 1).

n	s	e	$n \oplus s \oplus e$	w_+	w_-
0	0	0	0	0	0
0	0	1	1	0	1
0	1	0	1	0	1
0	1	1	0	1	1
1	0	0	1	0	0
1	0	1	0	1	0
1	1	0	0	1	0
1	1	1	1	1	1

Table 1: Local truth table. Addition and subtraction coincide exactly when $e = s$.

4 Conclusion

What looks like a tiling puzzle is really a small computer. The correct result is the only assembly that fits. Computiles show that binary arithmetic can be embedded in geometry and enforced by matter itself. Will someone provide 3D printable models?